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General Equilibrium Analysis: Methodology and Modeling Tools

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Applied General Equilibrium Models

Models provide a stylized story of how markets work and the nature of agent interactions:

1. Theory of the consumer (demand), including intertemporal choice.
2. Production and cost theory (supply), possibly based on (bottom-up) activity analysis – engineering estimates of cost functions.
3. The neoclassical paradigm: individual elements of the economy (consumers, firms, workers) are rational agents with objectives which can be expressed as quantiative functions to be optimized subject to constraints.
4. The theory underlying approach is dated and has not been a core element of graduate economics classes for over 20 years.

In Defense of a Neoclassical Approach

1. Versatile: can be extended to take into account many aspects which are often assumed to be ignored: risk and uncertainty, technological details, expectations.
2. Can be both calibrated and estimated. Hence, it is possible to formulate a model which matches both with the current economic statistics (supply and demand) and which matches historical evidence about the responsivenss of quantity to price.
3. Approach can be consistant with the principal of Occam's Razor: "A scientific theory should be as simple as possible, but no simpler."
4. Modesty is warranted: existence of model results should be a *necessary but not a sufficient* condition as justification for a particular policy proposal.

Positive versus Normative

- In economics, *positive* statements are factual statements whose truth or falsehood can be verified by empirical study, logic or careful computer programming.
- Conversely, *normative* statements involve value judgments and cannot be verified by empirical study or logic.

Economic equilibrium models are typically used to investigate logical implications of specific assumptions. Model results may provide the basis for normative conclusions.

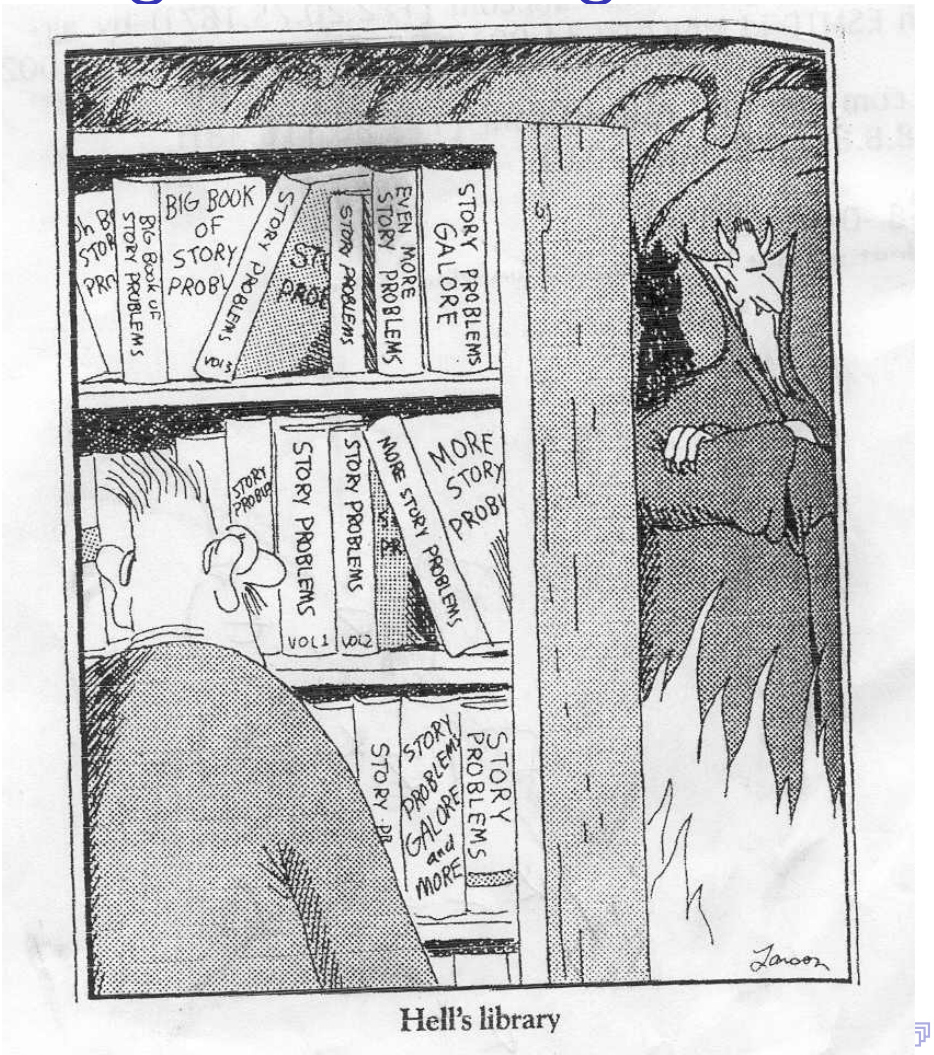
The bad news...

To do applied general equilibrium modelling:

> *"...one has to be familiar with general equilibrium theory, to be able to program, to be familiar with data and be able to manipulate and convert it into a model admissible form, to be conversant with literature estimates of key parameters, to have a clear sense of policy issues and institutional structure, and to be able to interpret results"*

[John Whalley1986]

The CGE Challenge: Translating Stories into Equations



Hell's library

The good news...

- advent of special purpose software has lessened the need for programming skills - e.g. GAMS
- teams of researchers have developed excellent and well-documented models - e.g. GAMS/GTAP models
- Over the past decade CGE has become more powerful and more easily accessible
- Short courses provide hands-on experience with "real" data and models.

Computing software

- GAMS is a general purpose scripting language specifically designed to assist the implementation of large optimization and economic models
- the user can write out the model in a transparent fashion, even with no previous programming experience

The Impetus for GAMS (Alex Meeraus)

"GAMS's impetus for development arose from the frustrating experience of a large economic modeling group at the World Bank. In hindsight, one may call it a historic accident that in the 1970s mathematical economists and statisticians were assembled to address problems of development. They used the best techniques available at that time to solve multi sectoral economy-wide models and large simulation and optimization models in agriculture, steel, fertilizer, power, water use, and other sectors. Although the group produced impressive research, initial success was difficult to reproduce outside their well functioning research environment. The existing techniques to construct, manipulate, and solve such models required several manual, time-consuming, and error-prone translations into different, problem-specific representations required by each solution method."

MPSGE: A Mathematical Programming System for General Equilibrium Analysis

- Model representation tool for a specific class of economic models: Arrow-Debreu general equilibrium.
- Incorporates facilities for automatic calibration of cost and expenditure functions
- Model input is *tabular* (non-algebraic) and follows the broad schematic structure of the MPS format for general equilibrium models
- Provides routines for providing demand and supply functions (price-responsive netputs for production sectors and excess demands for consumers) and *analytic Jacobians*.
- Provides background error checks related to model consistency

MPSGE Historical Timeline

1981 Lars Mathiesen spends sabbatical at the Stanford OR Department with the research objective of implementing Newtons method for generalized equations [Josephy (1979)]

1982 A pilot implementation of the nonlinear complementarity solver MILES is completed, based in Tomlin's LCPL code for Lemke's algorithm and Saunders' LUSOL (the sparse matrix factorization code from MINOS)

1982 A trade policy research project, *Market Prospects*, is undertaken at NHH in Bergen. A central element of the project is a global Heckscher-Ohlin trade model, VEMOD. Project participants included Victor Norman, Agnar Sandmo, Lars Mathiesen, Terje Hansen, Terje Lensburg, and Erling Stigum.

1983 A standardized set of routines for representing nested CES functions is implemented to help with the ongoing formulation and reformulation of VEMOD.

1984 A pilot implementation of MPSGE is presented at TIMS XXVI, June 17-21, 1984.

A 2x2 Closed Economy Model in MPSGE

```
$MODEL:M2X2

$SECTORS:
        X       ! Capital-intensive activity level
        Y       ! Labor-intensive activity level

$COMMODITIES:
        PX      ! Price of capital-intensive goods
        PY      ! Price of labor-intensive goods
        PL      ! Wage rate
        RK      ! Return to capital

$CONSUMERS:
CONS    ! Representative agent

$PROD:X S: 1
        O:PX    Q: 100  A:CONS  T:-0.1
        I:PL    Q: 40
        I:RK    Q: 60

$PROD:Y S: 1
        O:PY    Q: 100
        I:PL    Q: 60
        I:RK    Q: 40

$DEMAND:CONS    S: 1
        D:PX    Q: 100
        D:PY    Q: 100
        E:PL    Q: 100
        E:RK    Q: 100

$SOLVE
```

Ukraine Input-Output Table (2017)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	*Таблиця "витрати-випуск" України за 2017 рік в основних цінах*													
2	**ТАБЛИЦЯ ВИТРАТИ-ВИПУСК В ОСНОВНИХ ЦІНАХ**													
3													*(млн.грн)*	
4			**Проміжне споживання**											
5														
6														
7		Код КВЕД	Сільське, лісове та рибне господарство	Добування кам'яного та бурого вугілля	Добування сирої нафти та природного газу	Добування металевих руд, інших корисних копалин та розроблення кар'єрів; надання допоміжних послуг у сфері добувної промисловості та розроблення кар'єрів	Виробництво харчових продуктів; напоїв та тютюнових виробів	Текстильне виробництво, виробництво одягу, шкіри та інших матеріалів	Виробництво деревини, паперу; поліграфічна діяльність та тиражування	Виробництво коксу та коксопродуктів	Виробництво продуктів нафтоперероблення	Виробництво хімічних речовин і хімічної продукції	Виробництво основних фармацевтичних продуктів фармацевтичних препарати	Виробництво гумових і пластмасових виробів
8			A01-A03	B05	B06	B07-B09	C10-C12	C13-C15	C16-C18	C19.1	C19.2	C20	C21	C22
9														
10	Сільське, лісове та рибне господарство	A01-A03	165656	167	72	64	133751	470	8582	4	3	966	225	1
11	Добування кам'яного та бурого вугілля	B05	576	1427	1	30	1801	2	131	33789		54		2
12	Добування сирої нафти та природного газу	B06	4534	21	5983	3833	15550	318	2236	243	34135	14531	208	11
13	Добування металевих руд, інших корисних копалин та розроблення кар'єрів; надання допоміжних послуг у сфері добувної промисловості та розроблення кар'єрів	B07-B09	415	535	1692	12553	2131	13	53	2	314	673	3	4
14	Виробництво харчових продуктів; напоїв та тютюнових виробів	C10-C12	4291	16	113	31	50381	13	21	2		129	43	
	Текстильне виробництво, виробництво одягу, шкіри													

Notation

1. Sets

   g Goods
   s Sectors (=goods + components of final demand)

2. Components of final demand

   hcon Household consumption
   npsh Non-profit institutions serving households
   inv Gross fixed capital formation
   gov General government

IOT Representation in MPSGE

```
$model:UKR

$sectors:
        Y(s)            ! Sectoral production (sector s)
        X(g)            ! Export-domestic allocation (commodity g)
        ID(g,s)         ! Intermediate demand (commodity g to sector s)

$commodities:
        PY(s)           ! Output price
        PID(g,s)        ! Shadow price of intermediate demand
        PD(g)           ! Domestic price
        RK(g)           ! Return to capital
        PL              ! Wage rate
        PFX             ! Relative price of foreign exchange

$consumers:
        RA              ! Representative agents
```

Static Model for Ukraine (Intermediate Demand)

An elasticity of substitution $\sigma = 4$ characterizes the substitution elasticity between domestic and imported intermediate inputs of good g to sectors s:

$$ID_{gs} = \psi_{gs} \left(\beta_{gs} D_{gs}^{\gamma} + (1 - \beta_{gs}) D_{gs}^{\gamma} \right)^{1/\gamma}$$

where $\gamma = 1 - 1/\sigma = 0.75$

```
$prod:ID(g,s)$id0(g,s)   s:4
        o:PID(g,s)                q:id0(g,s)
        i:PD(g)                    q:dd0(g,s)
        i:PFX                      q:md0(g,s)
```

Static Model for Ukraine (Production)

i. Factor inputs are Cobb-Douglas:

$$Y_s = \Gamma_s L_s^{\theta_s} K_s^{1-\theta_s}$$

ii. Intermediate inputs are Leotief:

$$ID_{gs} = a_{gs} Y_s$$

```
$prod:Y(s)  s:0   va:1
o:PY(s)     q:y0(s)                                a:RA    t:ty(s)
i:PID(g,s)  q:id0(g,s)     p:(1+tx0(g,s))  a:RA    t:tx(g,s)
I:PL        q:ld0(s)        va:
I:RK(s)     q:kd0(s)        va:
```

Static Model for Ukraine (Allocation of Exports)

An elasticity of transformation $\eta = 4$ characterizes the transformation elasticity between goods produced for the domestic and export markets:

$$Y_g = \phi_g \left(\alpha_g E_g^{\rho} + (1 - \alpha_g) D_g^{\rho}\right)^{1/\rho}$$

where $\rho = 1 + 1/\eta = 1.25$

```
$prod:X(g)        t:4
o:PFX    q:e0(g)  a:RA     t:te(g)    p:(1-te0(g))
o:PD(g)  q:d0(g)
i:PY(g)  q:y0(g)
```

Static Model for Ukraine (Endowments and Demand)

```
$demand:RA
        d:PY("hcon")      q:y0("hcon")

        e:PY("npsh")      q:(-y0("npsh"))
        e:PY("inv")       q:(-y0("inv"))
        e:PY("gov")       q:(-y0("gov"))

        e:PD(g)           q:(-dstk0(g))
        e:PFX             q:(-sum(g,mstk0(g)))

        e:PFX             q:vb0
        e:PL              q:le0
        e:RK(g)           q:kd0(g)
```

Calibration

The model provides an idealized representation of the interaction of markets and various agents. Adaptation of the story to match a given set of potentially inconsistent data requires *calibration*. To do this we need to impose benchmark equilibrium conditions on the input data:

1. Zero profit – any sector operated at nonzero intensity breaks even.
2. Market clearance – supply equals demand for every commodity and primary factor.
3. Income balance – the value of factor income equals the value of expenditure (including household consumption, public expenditure and investment).

Model Parameters

3. Fixed benchmark values

$\hat{vb}$	Current account deficit
$\hat{dstk}_g$	Domestic stock change
$\hat{mstk}_g$	Change in stock of imports

4. Target benchmark values

$\hat{y}_s$	Sectoral output
$\hat{dd}_{gs}$	Domestic intermediate demand
$\hat{md}_{gs}$	Imported intermediate demand
$\hat{ld}_s$	Labor demand
$\hat{kd}_s$	Capital demand
$\hat{d}_g$	Domestic supply of commodity g
$\hat{e}_g$	Export of commodity g

Matrix Balancing Objective

Objective function defined in terms of the norm applied to deviations from target values ($\mathcal{N}(\delta)$). When:

$\mathcal{N}(\delta) = \delta^2$ we minimize *least squares* (the L2 norm).

$\mathcal{N}(\delta) = |\delta|$ we minimize *absolute differences* (the L1 norm).

$$
\begin{aligned}
\min Z = \quad & \textstyle\sum_s \mathcal{N}(\bar{y}_s - \hat{y}_s) + \\
& \textstyle\sum_{gs} \mathcal{N}(\bar{id}_{gs} - \hat{dd}_{gs}) + \\
& \textstyle\sum_{gs} \mathcal{N}(\bar{md}_{gs} - \hat{md}_{gs}) + \\
& \textstyle\sum_s \mathcal{N}(\bar{ld}_s - \hat{ld}_s) + \\
& \textstyle\sum_s \mathcal{N}(\bar{kd}_s - \hat{kd}_s) + \\
& \textstyle\sum_g \mathcal{N}(\bar{d}_g - \hat{d}_g) + \\
& \textstyle\sum_g \mathcal{N}(\bar{e}_g - \hat{e}_g)
\end{aligned}
$$

Zero Profit Conditions

- Zero profit for Y_s:

$$\bar{y}_s(1 - ty_s) = \sum_g \overline{id}_{gs}(1 + tx_{gs}) + \overline{ld}_s + \overline{kd}_s$$

- Zero profit for X_g:

$$\bar{y}_g = \bar{d}_g + \bar{e}_g(1 - te_g)$$

- Zero profit for ID_{gs}:

$$\overline{id}_{gs} = \overline{dd}_{gs} + \overline{md}_{gs}$$

Market Clearance Conditions

- Market Clearance for PD_g:

$$\bar{d}_g = \sum_s \overline{dd}_{gs} + \hat{dstk}_g$$

- Market Clearance for PL:

$$\overline{le} = \sum_s \overline{ld}_s$$

- Market Clearance for PFX:

$$\hat{vb} + \sum_g \bar{e}_g = \sum_g \hat{mstk}_g + \sum_{gs} \overline{md}_{gs}$$